WITH EFFECT FROM 2012 – 2013 ADMITTED
BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS SECOND SEMESTER
PAPER – I :: COMPLEX ANALYSIS

Max. Marks: 70
Time: Three hours

There are ten questions and each carrying 14 marks .One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT - I

Limits and continuity, Complex differentiability, Cauchy – Reiman equations, Exponential, Logarithmic, Trigonometric and hyperbolic functions.
(Chapter 3 of Ref. (1))

UNIT - II

Line integral, Local and Global Primitives, Cauchy – Goursat theorem, Cauchy's theorem.
Winding number, Cauchy's integral formulae, Cauchy's estimate, Liouville's theorem, fundamental theorem of Algebra, Morera's theorem .
(Chapters 4 and 5 of Ref. (1)

UNIT – III

Infinite series, Series of functions and uniform convergence, power series.
Taylor series, Zeros of analytic functions, Laurent series, Singularities .
(Chapters 6 and 7 of Ref. (1))

UNIT – IV

Resdue theorem and evaluation of standard integrals.
(Sections 8.1 and 8.2 of Chapter 8 of Ref. (1))

UNIT – V

Logaritmic residue, Rouche's theorem,
Conformal mappings, Linear transformation, cross ratio symmetry, Schwarz – Christoffel transformation.
(Section 8.3 of Chapter 8 and Chapter 9 of Ref. (1))

References

1. The Elements of Complex Analysis, B. Choudhary, Wiley Eastern Ltd., 1983.

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BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS SECOND SEMESTER
PAPER – II :: CLASSICAL MECHANICS

Max. Marks: 70
Time: Three hours

There are ten questions and each carrying 14 marks .One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT – I

Moments and Products of Inertia: Moments and Products of inertia evaluation in standard cases, Theorem of parallel and Perpendicular axes, Momental ellipsoid.
(Sections 144 to 151 of Ref. (1))

UNIT – II

Equimomential systems, principal axes at a point of its length , D'Alembert's Principle**,** The general equations of motion of rigid body under finite and impulsive forces.
(Sections 152 to 157 and 160 to 166 of Ref. (1))

UNIT – III

Motion about a fixed axis of rigid body, compound pendulum, Reacions of axis of rotation, Sample equivalent pendulums.
(Sections 168 to 176 and 179 to 181 of Ref. (1))

UNIT – IV

Motion in two dimensions: Motion in two dimensions of a rigid body under finite and impulsive forces.
(Sections 187, 189 to 191, 194 to 202 and Sections 204, 205, 207 of Ref. (1))

UNIT – V

Euler's Equations of Motion: Euler's dynamical equations of motions of a rigid body about a fixed point under finite and impulsive forces, Eulerien angles, Euler's geometrical equations.
(Sections 9.2, 9.3 and 10.13 of Ref. (2))
***For problems in all units refer Ref.(3)**

References

1. An Elementary Treatise on the Dynamics of a particle and of Rigid Bodies by S.L.Loney, Macmillan Company of India Ltd., Metric Edition (1976)
2. "Text Book of Dynaics" by F. Chorlton, CBS Publishing and distributions, Second edition.
3. "Dynamics of a rigid bodies" by B.S. Tyagi, Brahmanand and Bhu Dev Sharma, Kedharnadh Ramnadh Publ., Meerut.

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KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS SECOND SEMESTER
PAPER – III :: PARTIAL DIFFERENTIAL EQUATIONS

Max. Marks: 70
Time: Three hours

There are ten questions and each carrying 14 marks .One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I

First Order Partial Differential equations: Curves and Surfaces, Genesis of first order partial differential equations , Classification of Integrals, Linear equations of the first order, Pfaffian Differential equations, Compatible systems. Charpit's method. (Sections 1.1 to 1.7 of Chapter 1 of [1]).

UNIT-II

Jacobi's Method, Integral surfaces through a given curve, Second order Partial differential Equations: Genesis of Second Order Partial Differential Equations. Classification of Second Order Partial differential equations (Sections 1.8 & 1.9 of Chapter 1 of [1], Sections 2.1 to 2.2 of Chapter 2 of [1]).

UNIT-III

One Dimensional Waves equation, Vibrations of an infinite string, Vibrations of a semi infinite string. Vibrations of a string of Finite Length, Riemann's Method, Vibrations of a string of finite length (Method of separation of variables.)
(Sections 2.3.1 to 2.3.5 of Chapter 2 of [1]).

UNIT-IV

Laplaces Equation: Boundary value problems, Maximum and Minimum principles, The Cauchy problem, The Dirichlet problem for the upper Half plane, The Neumann problem for the upper Half plane, The Dirichlet Interior problem for a circle, The Dirichlet Exterior problem for a circle, The Neumann Problem for a circle, The Dirichlet problem for a Rectangle..(Sections 2.4.1 to 2.4.9 of Chapter 2 of [1]).

UNIT-V

Harnacks Theorem, Laplaces Equation – Green's Function. The Dirichlet problem for a Half plane, The Dirichlet problem for a circle, Heat conduction- Infinite Rod case, Heat conduction – Finite Rod case, Duhamel's principle, Wave equation, Heat Conduction Equation. (Sections 2.4.10 to 2.4.13, 2.5.1 to 2.5.2, 2.6.1 to 2.6.2 of Chapter 2 of [1]).

PRESCRIBED BOOK: [1] **T.AMARANATH, An Elementary course in Partial Differential Equations, Second Edition, Narosa Publishing House, 2003**

REFERENCE BOOKS: [2]. SNEEDON IAN, Elements of Partial Differential Equations, Tata Mc Graw Hill, 1987

[3]. K. SANKARA RAO, Introduction to Partial Differential Equations, PHI, 2003

WITH EFFECT FROM 2012 – 2013 ADMITTED
BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS SECOND SEMESTER
PAPER – IV :: NUMERICAL METHODS WITH C

Max. Marks: 70
Time: Three hours

There are ten questions and each carrying 14 marks .One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT – I
C – Basics
C – Character set
Data types
Variables
Constants
Expressions
Structure of C program
Operators and their precedence and Associativity
Basic input and output statements
Control structures
Simple programs in c using all the operators and control structures
Functions
Concept of a function
Parameters and how they are passed
Automatic Variables
Recursion
Scope and extent of variables
Writing programs using recursive and non – recursive functions

(1.4,1.7,1.11,1.12 of Chapter 1, 2.2,2.3,2.4 of Chapter 2 , 3.1,3.2,3.3 of Chapter 3 & 5.1, 5.2,5.3 of Chapter 5 of [1])

UNIT – II
Arrays and Strings
Single and multidimensional Arrays
Character array as a string
Functions on strings, Writing C Programs using arrays and for string manipulation.
Pointers
Pointers declarations
Pointers expressions
Pointers as parameters to functions
Pointers and Arrays
Pointer arithmetic
Structures & Unions
Declaring and using structures

Operations on structures
Arrays of structures
User defined data types
Pointers to Structures
(4.1 to 4.6 of Chapter 4, 6.1 to 6.8 of Chapter 6 , Chapter 9 & Chapter 10 of [1])

UNIT-III

Interpolation and Approximation: Introduction, Lagrange and Newton Interpolations, Finite difference Operators, Interpolating polynomials using finite differences, Hermite Interpolations. (Section 4.1 to 4.5 of chapter 4 of [2]).

UNIT-IV

Numerical Differentiation and Integration: Introduction, Numerical differentiation ,Numerical integration, Methods based on Interpolation, Methods based on Undetermined Coefficients, Composite Integration Methods.
(Sections 5.1, 5.2, 5.6, 5.7,5.8, 5.9 of chapter 5 of [2])

UNIT-V

Ordinary Differential Equations: Introduction, Numerical Method, Single step Methods, Multistep methods.(sections 6.1 to 6.4 of chapter 6 of [2])

PRESCRIBED BOOKS: [1] AJAY MITTAL, C Programming a practical approach, Pearson,.
[2] M.K.JAIN S.R.K. IYANGAR AND R.K. JAIN, Numerical Methods for Scientific and Engineering Computation, Third edition, New Age International (p) Limited, New Delhi, 1997.

REFERENCE BOOKS: 1.P.C. BISWAL, Numerical Analysis, PHI, 2008
2. V. RAJA RAMAN, Computer Oriented Numerical Methods, Third Edition, PHI

WITH EFFECT FROM 2012 – 2013 ADMITTED
BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS SECOND SEMESTER
PAPER – V :: GRAPH THEORY

Max. Marks: 70
Time: Three hours

There are ten questions and each carrying 14 marks .One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I

Introduction: What is a Graph, Finite and Infinite graphs, Incidence and degree, Isolated Vertex, Pendant Vertex and Null Graph
Paths and circuits: Isomorphism, Subgraphs , a puzzle with multi colored cubes. walks, Paths and Circuits, connected graphs, Disconnected graphs, Components, Euler graphs , Operations on graphs, More on Euler graphs, Hamiltonian paths and circuits, Travelling – Salesman Problem (Chapters 1 and 2 of [1]).

UNIT-II

Trees and Fundamental Circuits: Trees , some properties of trees , pendant Vertices in a tree, distances and centers in a tree, rooted and binary trees, on Counting trees, spanning trees, fundamental circuits, finding all spanning trees of a graph , spanning trees in a weighted Graphs. (Chapter 3 of[1].)

UNIT-III

Cut sets and Cut –vertices: Cut sets, Some Properties of a Cut Set, All cut sets in a Graph, Fundamental circuits and cut sets, connectivity and separability, network flows, 1-isomorphism, 2- isomorphism. (Chapter 4 of [1])

UNIT-IV

Planar and dual graphs: Combinatorial Vs Geometric graphs , Planer graphs, Kuratowski's two graphs , Different representations of a planar graph , Detection of planarity, Geometric dual .[Sections 1 to 6 of Chapter 5 of [1])

UNIT-V

Vector spaces of a graph: Sets with one operation , Sets with two operations, Modular arithmetic and Galois field, Vectors and Vector spaces, Vector space associated with a graph , Basis vectors of graph ,circuits and cut-set sub spaces. (Sections 1 to 7 of Chapter 6 of [1])

PRESCRIBED BOOK: [1]**NARSINGH DEO, Graph theory with applications to Engineering and Computer Science, Prentice Hall of India Pvt., New Delhi, 1993.**

REFERENCE BOOK: BONDY J.A AND U.S.R. MURTHY, Graph Theory with Applications, North Holland

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

Second Semester

Applied Mathematics

Paper I – Complex Analysis

Time : Three hours   Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

Unit-I

(1) (a) Let $u(x,y)$ and $v(x,y)$ be real valued functions defined on a region Ω and suppose that $u(x, y)$ and $v(x,y)$ have continuous partial derivatives. Then prove that $f: \Omega \to C$ defined by $f(z)=u(x,y)+i\,v(x, y)$ is analytic iff u and v satisfy the C – R equations.

(b) Define an analytic function.Find the analytic function whose real part is $u(x,y) = e^{x} \cos y$.

(OR)

(2) (a) Define the harmonic function. Prove that the harmonic function satisfies the differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$

(b) State and prove chain rule for analytic functions.

Unit-II

(3) (a) State and prove Cauchy's – Goursat theorem.

(b) Evaluate $\int_{1}^{2+i} (x^2 + iy)dz$ along (i) The line $y=x-1$ (ii) the line $y=(x-1)^2$.

(OR)

(4) (a) State and prove the Cauchy's integral formula.

(b) State and derive Cauchy's estimate. Deduce Liouville's theorem.

Unit-III

(5) (a) If $\{f_n(z)\}$ is a Cauchy sequence of functions on E then show that $\{f_n(z)\}$ converges uniformly on E.

(b)) Show that the series $1+\frac{1}{2^2}\left(\frac{z-1}{z+1}\right)+\frac{1}{3^2}\left(\frac{z-1}{z+1}\right)^2+\frac{1}{4^2}\left(\frac{z-1}{z+1}\right)^3+\cdots\cdots$

converges for Re(z) ≥ 0 and diverges for Re(z) < 0.

(OR)

(6) (a) Find the Laurent expansion of $f(z)=\frac{1}{z(z-1)(z-2)}$ in (i) ann (0,0,1)

(ii) ann (0,1,2).

(b) State and prove Laurent's theorem.

Unit-IV

(7) (a) State and prove Cauchy residue theorem.

(b) Evaluate (i) $\int_0^\infty \frac{\log x}{(1+x^2)^2}dx$ (ii) $\int_0^{\pi/2} \frac{d\theta}{1+\sin^2\theta}dx$

(OR)

(8) (a) Show that $\int_0^\infty \frac{\log x}{1+x^2}dx=0$ by residue theory.

(b) Evaluate $\int_0^\infty \frac{1}{1+x^4}dx$

Unit-V

(9) (a) Define Mobius transformation. Show that Mobius transformation leaves cross – ratio invariant.

(b) Show that a Mobius transformation takes circles onto circles.

(OR)

(10)(a) State and prove Rouches theorem.

(b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z=1|$ and $|z=2|$.

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

Second Semester

Applied Mathematics

Paper II – Classical Mechanics

Time : Three hours Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

Unit-I

(1) (a) If the moments and products of inertia of a body about three perpendicular and concurrent axes are known, find the M.I. about any other axis through the meeting point .

(b) Find the moment of inertia of a uniform solid sphere of mass M and radius "a" about its diameter and hence find M.I. of it about a tangent.

(OR)

(2) (a) Find the M.I. of a truncated cone about its axis, the radii of its ends being a and b.

(b) Define a momental ellipsoid at a point O of a rigid body.

Show that the momental ellipsoid at the centre of an elliptic plate is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+z^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=$ constant.

Unit-II

(3) Derive the equimomental system for a triangle of mass M and hence find the M.I. of the triangle about its one side.

(OR)

(4) (a) State and prove D'Alembert's principle.

(b) At the vertex C of triangle ABC ,with right angle at C ,show that the principal axes are perpendicular to the plane and two others are inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\left(\frac{ab}{a^2-b^2}\right)$.

Unit-III

(5) (a) Drive the equation of motion of compound pendulum and hence find the time of a complete small oscillation.

(b) A solid homogeneous cone of height *'h'* and vertical angle *2a* oscillates about a horizontal axis through its vertex. Find the length of the simple equivalent pendulum.

(OR)

(6) (a) A uniform rod of mass m and length $2a$ can turn freely about one end which is fixed. Find its motion if it starts rotating with angular velocity ω from the position in which it hangs vertically.

(b) A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position. Show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle of 45^0 to the vertical and that the vertical strain is then $11/8$ times the weight of the rod.

Unit-IV

(7) (a) Find the K.E. of body moving in two dimensions.

(b) A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Find its motion .

(OR)

(8) (a) A uniform rod is held at an inclination a to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ. If it be then released, show that it will commence to slide if $\mu < \frac{3\sin\alpha\cos\alpha}{1+3\sin^2\alpha}$.

(b) A uniform rod *AB* of length *2a*, is lying on smooth horizontal plane and is struck by a horizontal blow, of impulse *p*, in a direction perpendicular to the rod at a point of distant *b* from its center. Find the motion.

Unit-V

(9) (a) Derive Euler's dynamical equations of motion of rigid body about a fixed point under finite forces .

(b) A body under the action of no forces moves so that the resolved part of its angular velocity about one of the principal axes at the center of gravity is constant. show that the angular velocity of the body must be constant.

(OR)

(10) (a) Find the locus of invariable line.

(b) A body moves about a point O under no forces, the principal moments of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n, \omega_2 = 0, \omega_3 = n$ about the corresponding principal axes. Show that at any latter time t,

$\omega_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$ and that the body ultimately rotates about its mean axis.

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

Second Semester

Applied Mathematics

Paper III – Partial Differential Equations

Time : Three hours   Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

UNIT-I

1. (a)Define General Integral and Find the General Solution of $2x(y+z^2)\,p + y(2y+z^2)q = z^3$
   (b)Show that the Necessary and Sufficient condition for the Pfaffian differential equation $X.\,dr = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$ to integrable is $X.\,\text{Curl}\,X = 0$

(OR)

2. (a)Find the integral of $yz\,dx + (x^2y - zx)\,dy + (x^2z - xy)\,dz = 0$.
   (b)Define Complete Integral and Find a complete integral of $p^2x + q^2y = z$

UNIT-II

3. (a) Solve $xu_x + yu_y = (u_z)^2$ by Jacobi's Method.
   (b) Find the Integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 1$, $y_0(s) = 0$ and $z_0(s) = s$.

(OR)

4. (a) Find the Integral surface of the equation $p^2x + pqy = 2pz + x$, passing through the line $y=1$, $x=z$
   (b) Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form.

UNIT-III

5. (a) Find the d'Alembert's Solution for the problem of Vibrations of a string of infinite length.
   (b) Prove that for the equation $u_{xy} + \frac{1}{4} u = 0$, the Riemann function is
   $v(x,y,\alpha,\beta) = J_0(\sqrt{(x-\alpha)(y-\beta)})$

(OR)

6. (a) Solve the Problem of vibrations of a string of finite length using the method of separation of variables.
   (b)Show that the solution of the problem of vibrations of string finite length is unique.

UNIT-IV

7. Solve the Dirichlet interior problem for a circle

(OR)

8. (a) Solve the Dirichlet problem for a Rectangle.
   (b) Solve the Neumann problem for the upper Half Plane.

UNIT-V

9. (a)State and Prove Harnack's Theorem.
   (b) Solve the Dirichlet problem for a Half plane using Green's function

(OR)

10. (a) Solve Heat Conduction problem in a rod of infinite length.
    (b) Solve $u_{tt} - c^2 u_{xx} = F(x,t)$, $0< x < l$, $t > 0$

    $U(x,0)= f(x)$,

    $U_t(x,0)= g(x)$,

    $U(0,t) = u_t(l,t) =0$,

    by making use of Duhamel's Principle.

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

Second Semester

Applied Mathematics

Paper IV – Numerical Methods With C

Time : Three hours Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

UNIT -I

1 **(a) Write notes on data types supported by C.**

(b) Write a program to find standard deviation of a given sequence.

(OR)

2 **(a) write a note on operator precedence and associativity.**

(b) Write a program to check given string is porindrome or not.

UNIT -II

3 **(a) Write notes on storage classes in C.**

(b) Write a program to find n fibonocci numbers using functions.

(OR)

4 **(a) Distinguish between structures and unions. Define pointers.**

(b) Write a program to find the sum of complex numbers.

UNIT- III

5 **(a) Derive Newton's divided difference interpolation formula.**

(b) Find the unique polynomial of degree 2 or less such that f(0) =1, f(1)=3, f(3)= 55 using the Newton-Divided Difference interpolation.

(OR)

6 **(a) Derive Lagrange's interpolation formula.**

(b) Certain corresponding value of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871) find $\log_{10} 301$

UNIT –IV

7 (a) Derive Gauss- Legendre three point formula for numerical integration.

(b)Evaluate the integral

$$\mathbf{I} = \int_{-1}^{1} (1-x^2)^{3/2} \cos x dx$$

Using Gauss- Chebyshev three – point formula.

(OR)

8 Find an approximate value of $\mathbf{I} = \int_0^1 \frac{dx}{1+x}$ **using**

(a) Trapezoidal rule (b) Simpson's rule . Obtain a bound on the error.

UNIT -V

9 (a) Derive a formula for Euler's method to solve an initial value problem and it's error.

(b) Solve the I.V.P.

$u' = -2tu^2$, $u(0)=1$

with h=0.2 on the interval $[0,1]$ **using the back ward Euler method.**

(OR)

10 (a) Derive second order Runge-Kutta method for solving initial value problem.

(b) Solve the system

$u' = -3u + 2v$

$v' = 3u - 4v$, $u(0) = 0$, $v(0) = 0.5$

with h= 0.2 on the interval $[0,0.4]$ **using the Euler- Cauchy method.**

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

Second Semester

Applied Mathematics

Paper V – Graph Theory

Time : Three hours Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

UNIT-I

(1) (a) If graph G has exactly two vertices of odd degree, then show that there must be a path joining these two vertices.

(b) Show that a simple graph with n vertices and k components can have atmost ½(n-k)(n-k+1) edges.

(OR)

(2) (a) Prove that a connected graph G is Euler graph if and only if all vertices of G are of even degree.

(b) Define a Hamilton circuit. Prove that a complete graph with n vertices have ½(n-1) edge-disjoint Hamilton circuits, if n is an odd number ≥3.

UNIT-II

(3) (a) Define a tree. Prove that there is one and only one path between every pair of vertices in a tree.

(b) Prove that every tree contains at least two pendent vertices.

(OR)

(4) (a) Define a spanning tree. Show that every connected graph has at least one spanning tree.

(b) Define a shortest spanning tree. Show that a spanning tree is a shortest spanning tree if and only if there exists no other spanning tree at a distance of one from T whose weight is smaller than that of T

UNIT-III

(5) (a) Define a cut set. Show that every cut set in a connected graph G must contains at least one branch of every spanning tree of G

(b)Show that every circuit has an even number of edges in common with any cut set.

(OR)

(6) (a) Define a cut vertex. Show that a vertex v in a connected graph G is a cut vertex if and only if there exists two vertices x and y in G such that every path between x and y posses through v

(b) Define the edge connectivity of a graph. Show that the edge connectivity of a graph can never exceed the degree of the vertex with smallest degree in G.

UNIT-IV

(7) (a) Define a planar graph. Show that the complete graph with five vertices is non planar.

(b) Show that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

(OR)

(8) (a) State and prove Euler's formula.

(b) How to detect the given graph is planar or non- planar.

UNIT-V

(9) (a) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge disjoint unions of circuits.

(b) Prove that the set consisting of all the cut sets and the edge disjoint unions of cut sets in a graph G is an abelian group under the ring sum operation.

(OR)

(10) (a) Prove that in a graph G, W_G is a vector space.

(b) Prove that the set of all circuit vectors in W_G forms a sub space of W_S .