

WITH EFFECT FROM 2012 – 2013 ADMITTED
BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS FIRST SEMESTER
PAPER – I :: ALGEBRA

Max. Marks: 70
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT - I

Groups: Definition of a group, Some examples of group, Some preliminary lemmas, Subgroup, A counting principle, Normal subgroup and quotient groups, Homomorphisms, Automorphisms.

(Sections 2.1 to 2.8 of Chapter 2 of Ref.(1))

UNIT - II

Cayley' theorem, Permutation groups, Another Counting principle, Sylow's theorem, Direct products, Finite abelian groups.

(Sections 2.9 to 2.14 of Chapter 2 of Ref.(1))

UNIT – III

Rings: Definition and examples of Rings, Some special classes of Rings, Homomorphisms. Ideals and quotient rings, More ideals and quotient rings, The field of quotients of an integral domain, Euclidean rings, A particular Euclidean ring.

(Sections 3.1 to 3.8 of Chapter 3 of Ref.(1)).

UNIT - IV

Polynomial rings, polynomials over the rational field, polynomial rings over commutative rings. Extension Fields, The Transcendence of 'e'.

(Sections 3.9 to 3.11 of Chapter 3 and Sections 5.1 , 5.2 of Chapter 5 of Ref.(1))

UNIT - V

Roots of polynomials, more about roots, The elements of Galois theory, Solvability by radicals, Galois groups over the rationals.

(Sections 5.3 and 5.5 to 5.8 of Chapter 5 of Ref.(1))

References

1. I.N.Herstein, Topics in Algebra (Section edition, 1999), John Wiley and Sons (ASIA) Pre. Ltd. Singapore.

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BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS FIRST SEMESTER
PAPER – II :: REAL ANALYSIS

Max. Marks: 70
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT - I

Continuity: Limits of functions, continuous functions, continuity and compactness, continuity and connectedness. **(4.1 to 4.24 of Chapter 4 of Ref.(1))**

UNIT - II

Discontinuities, Monotonic functions, Infinite limits and limits at infinity.

Differentiation: The derivative of a real function, Mean value theorems, The continuity of derivatives **(4.25 to 4.34 of Chapter 4 and 5.1 to 5.12 and Corollary of Chapter 5 of Ref.(1))**

UNIT - III

L'Hospital's Rule, Derivatives of Higher order, Taylor's Theorem, Differentiation of vector valued functions.

The Riemann - Stieiltjes Integral: Definition and Existence of the integral **(5.13 to 5.19 of Chapter 5 and 6.1 to 6.11 of Chapter 6 of Ref.(1))**

UNIT - IV

Properties of the integral, Integration and differentiation, Integration of Vector - valued functions, Rectifiable curves. **Sequences and Series of Functions:** Discussion of main problem, Uniform convergence, Uniform convergence and continuity. **(6.12 to 6.27 of Chapter 6 and 7.1 to 7.15 of Chapter 7 of Ref.(1))**

UNIT-V

Uniform convergence and Integration, Uniform convergence and Differentiation, Equicontinuous families of functions, The Stone - Weierstrass theorem. **(7.16 to 7.27 of Chapter 7 of Ref.(1))**

References:

1. Principles of Mathematical Analysis (Third edition) by Walter Rudin, McGraw - Hill International Book Company.

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BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS FIRST SEMESTER
PAPER – III :: ORDINARY DIFFERENTIAL EQUATIONS

Max. Marks: 70
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT – I

Linear Equation of the first order: Linear equations of the first order, The equation $y' + ay = 0$, The equation $y' + ay = b(x)$, The general equations of the first order.

Linear Equations with constant coefficients: The homogeneous equation of order n , Initial value problems for n^{th} order equations.

(Chapter 1 of Ref.(1) and Section 7, 8 of Chapter 2 of Ref.(1))

UNIT – II

The non - homogeneous equation of order n , A special method for solving the non homogeneous equation.

Linear equations with variable coefficients: Initial value problems for the homogeneous equations, Solution of the homogeneous equations, The wronskian and linear independence.

(Sections 10,11 of Chapter 2 and Sections 1,2,3,4 of Chapter 3 of Ref.(1))

UNIT – III

Reduction of the order of a homogeneous equation, The non - homogeneous equation, Homogeneous equation with analytic coefficients (The proof for power series method is not needed), The Legendre equation.

(Sections 5,6,7,8 of Chapter 3 of Ref.(1))

UNIT – IV

Linear equations with regular singular points: The Euler equation, Second order equations with regular singular points, An example second order equation with regular singular points, The general case, The exceptional cases, The Bessel Equation.

(Sections 1, 2, 3, 4, 6 and 7 of Chapter 4 of Ref.(2))

UNIT – V

Existence Uniqueness and Continuation of solutions: Existence Uniqueness of solution of scalar differential equations, Existence theorem for system of differential equations, Differential and Integral inequalities.

(Section 1.1 to 1.5 of Chapter 1 of Ref.(2))

References

E.A.Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi (1987).

M.Rama Mohan Rao, Ordinary Differential Equations Theory and Applications, Affiliated East – West press Pvt.Ltd., New Delhi.

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BATCH AND SUBSEQUENT BATCHES

KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS FIRST SEMESTER
PAPER – IV :: TOPOLOGY

Max. Marks: 70
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT – I

Metric Spaces: definition and some examples, open sets, closed sets, convergence, completeness and Baire's theorem, continuous mappings
(Sections 9 to 13 of Chapter 2)

UNIT – II

Topological Spaces: The definition and some examples, Elementary concepts, Open bases and Open subspaces, Weak topologies
(Sections 16 to 19 of Chapter 3)

UNIT – III

Compactness: Compact spaces, Products of spaces, Tychonoff's theorem and locally compact spaces, Compactness for metric spaces, Ascoli's theorem.
(Sections 21 to 25 of Chapter 4)

UNIT – IV

Separation: T_1 - spaces and Hausdorff spaces, completely regular spaces and normal spaces, Urysohn's Lemma and the Tietze extension theorem.
(Sections 26 to 28 of Chapter 5)

UNIT – V

Connected spaces: connected spaces, The components of a space, Totally disconnected spaces.
(Sections 31 to 33 of Chapter 6)

Text Book:

Introduction to Topology and Modern Analysis by G. F. Simmons, Mc. Graw Hill book company, New York International Student Edition.

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KRISHNA UNIVERSITY POST GRADUATE CENTRE :: NUZVID
M.Sc., APPLIED MATHEMATICS FIRST SEMESTER
PAPER – V :: DISCRETE MATHEMATICS

Max. Marks: 70
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT –I

Logic : Computer Representation of Sets, Mathematical induction, Matrices, Logic, Tautology, Normal Forms, Logical Inferences, Predicate Logic, Universal Quantifiers, Rules of Inference.

(Chapter 1 of [3])

UNIT –II

Finite machines : Introduction, state tables and state diagrams, simple properties, Dynamics, Behavior and Minimization.

(Sections 5.1 to 5.5 of Chapter 5 of [1])

UNIT – III

Properties and Examples of lattices, Distributive Lattices, Boolean Algebras (Sections 1 to 3 of Chapter 1 of [2]).

UNIT –IV

Boolean polynomials, Ideals, filters and equations, Minimal forms of Boolean polynomials. (Sections 4,5,6 of Chapter -1)

UNIT –V

Application of lattices, switching circuits, Applications of switching circuits, More Applications of Boolean Algebras.

(Sections 7, 8, 9 of Chapter -2 of [2]).

PRESCRIBED BOOKS [1] JAMES L FISHER, "Application oriented Algebra" IEP, Dun-Downplay pub.1977.

[2] R.LIDL AND G. PILZ, Applied abstract algebra, Second Edition, Springer, 1998.

[3] RM. SOMASUNDARAM, Discrete Mathematical Structures, Prentice Hall of India, 2003

REFERENCE BOOK: J.P.TREMBLAY AND R. MANOHAR, Discrete Mathematical Structures with Applications to Computer Science, Tata Mc. Graw Hill, 2002.

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

First Semester

Applied Mathematics

Paper I – ALGEBRA

Time : Three hours

Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

Unit-I

(1)(a) If G is a finite group and H is a subgroup of G , then $o(H)$ is a divisor of $o(G)$.

(b) If H and K are subgroups of G and then $o(H) > \sqrt{o(G)}$, $o(K) > \sqrt{o(G)}$,
 $H \cap K \neq (e)$.

(OR)

(2) (a) State and prove Cauchy's theorem for abelian groups.

(b) Let Φ be a homomorphism of G onto G^1 with kernel K then show that G/K is isomorphic to G^1 .

Unit-II

(3) (a) State and prove Cayley's theorem.

(b) If $O(G) = p^2$, where p is a prime number, then show that G is abelian.

(OR)

(4) (a) If p is a prime number and $p^a / O(G)$, then show that G has a subgroup of order p^a .

(b) Let G be a group and suppose that G is the internal direct product of N_1, N_2, \dots, N_n . Let $T = N_1 \times N_2 \times \dots \times N_n$. Then show that G and T are isomorphic.

Unit-III

(5) (a) Show that a finite integral domain is a field.

(b) If R is a commutative ring with unit element and M is an ideal of R , then M is a maximal ideal of R if and only if R/M is a field.

(OR)

(6) Show that every integral domain can be embedded in a field.

Unit-IV

- (7) State and prove Division algorithm for polynomials over a field and prove that the polynomial ring $F[x]$ is a principal ideal ring.
(OR)
- (8) (a) If L is an algebraic extension of K and if K is an algebraic extension of F , then show that L is an algebraic extension of F .
(b) Show that the number “ e ” is transcendental.

Unit-V

- (9) State and prove Fundamental theorem of Galois theory.
(OR)
- (10) (a) Show that a polynomial of degree n over a field can have at most n roots in any extension field.
(b) If F is of characteristic zero and if a, b are algebraic over F then show that there exists an element c in $F(a, b)$ such that $F(a, b) = F(c)$

M.Sc.(Previous)DEGREE EXAMINATION,

Model paper

First Semester

Applied Mathematics

Paper II – REAL ANALYSIS

Time : Three hours

Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

Unit-I

- (1) (a) A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y .
- (b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y . Prove that $f(X)$ is compact.
- (OR)
- (2) (a) If f is a continuous mapping of a compact metric space into metric space Y then show that f is uniformly continuous on X .
- (b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X show that $f(E)$ is connected.

Unit-II

- (3) (a) Show that the set of points of discontinuities of a monotonic function on (a, b) is at most countable.
- (b) Let f be defined on $[a, b]$; if f has a local maximum at a point $x \in (a, b)$ and if $f^{-1}(x)$ exists, then show that $f^{-1}(x) = 0$.
- (OR)
- (4) (a) Let f be defined for all real x and suppose that $|f(x)-f(y)| \leq (x-y)^2$ for all real x and y . Prove that f is constant.
- (b) Let f be monotonically increasing on (a, b) . Then $f(x^+)$ and $f(x^-)$ exist at every point x of (a, b) and

$\sup_{a < t < x} f(t) = f(x^-) \leq f(x) \leq f(x^+) = \inf_{x < t < b} f(t)$ also show that, if $a < x < y < b$, then $f(x^+) \leq f(y^-)$.

Unit-III

- (5)(a) Suppose f is a continuous mapping of $[a, b]$ into \mathbb{R}^k and f is differentiable in (a, b) . Show that there exists $x \in (a, b)$ such that $|f(b) - f(a)| \leq (b-a) |f'(x)|$.
- (b) State and prove L'Hospital's rule.
- (OR)

- (6) (a) Show that a bounded function $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$, there exists a partition P of $[a, b]$ show that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.
- (b) If f is monotonic and α is continuous and monotonically increasing on $[a, b]$, then prove that $f \in R(\alpha)$.

Unit-IV

- (7) (a) If r' is continuous on $[a, b]$, then show that r is rectifiable and that $\Lambda(r) = \int_a^b |r'(t)| dt$.
- (b) State and prove fundamental theorem of integral calculus.
- (OR)

- (8) (a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
- (b) Suppose $f_n \rightarrow f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t \rightarrow x} f_n(t) = A_n$ ($n = 1, 2, \dots$). Then $\{A_n\}$ converge and $\lim_{t \rightarrow x} f(t) = \lim_{n \rightarrow \infty} A_n$.

Unit-V

(9) State and prove Stone –Weierstrass theorem

(OR)

(10) (a) Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then $\{f_n\}$ converges uniformly on $[a,b]$ to a function f and $f_n'(x) \rightarrow f'(x)$, $a \leq x \leq b$.

(b) If K is a compact metric space, if $f_n \in C(K)$ for $n= 1,2,3,\dots$ and if $\{f_n\}$ converges uniformly on K , then show that $\{f_n\}$ is equicontinuous on K .

M.Sc.(Previous)DEGREE EXAMINATION

Model paper

First Semester

Applied Mathematics

Paper III – ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

Unit-I

(1) (a) Find the general solution of the equation $y' + a(x)y = b(x)$, where $a(x)$ and $b(x)$ are continuous functions on some interval I .

(b) Let Φ be any solution of

$$L(y) = y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0$$

on an interval I containing a point x_0 , then for all x in I

$$|\Phi(x_0)| e^{-k|x-x_0|} \leq |\Phi(x)| \leq |\Phi(x_0)| e^{k|x-x_0|}, \text{ where } k = 1 + |a_1| + \dots + |a_n|.$$

(OR)

(2) (a) Find the solution φ of the equation $y' + (\cos x)y = e^{-\sin x}$ satisfying

$\varphi(\pi) = \pi$; and also show that φ has the property that $\varphi(\pi k) - \varphi(0) = \pi k$, where k is any integer.

(b) Prove that there exist n linearly independent solutions of

$$L(y) = y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0.$$

Unit-II

(3) (a) Solve $y'' + 4y = \sin 2x$ by annihilator method.

(b) Let $\Phi_1, \Phi_2, \Phi_3, \dots, \Phi_n$ be n solutions of

$L(y) = y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0$ on an interval I and let x_0 be any point in I . Then

$$W(\Phi_1, \Phi_2, \Phi_3, \dots, \Phi_n)(x) = \exp \left[- \int_{x_0}^x a_1(t) dt \right] W(\Phi_1, \Phi_2, \Phi_3, \dots, \Phi_n)(x_0).$$

(OR)

(4) (a) Find two linearly independent solution of the equation $(3x-1)^2 + (9x-3)-9y=0$ for $x > 1/3$.

(b) Obtain the general solution of the non homogeneous equation of order n by the method of variations of constants.

Unit-III

(5)(a) Find a second independent solution of $x^2 y^{11} - 7x y^1 + 15y = 0$ given that $\Phi_1(x) = x^3$ ($x > 0$) is a solution.

(b) Find two linearly independent series solution of Legendre equation $(1-x^2) y^{11} - 2xy^1 + \alpha(\alpha + 1)y = 0$, where α is a constant.

(OR)

(6) (a) If one solution of $x^2 y^{11} - 2y = 0$ on $0 < x < \infty$ is $\Phi_1(x) = x^2$. Find all solutions of $x^2 y^{11} - 2y = 2x-1$ on $0 < x < \infty$.

(b) Find two linearly independent power series solutions of the equation $y^{11} - x y^1 + y = 0$.

Unit-IV

(7) (a) Find all solutions of $x^2 y^{11} + xy^1 + 4y = 1$ for $|x| > 0$.

(b) Obtain two linearly independent solutions of $x^2 y^{11} + 3xy^1 + (1+x)y = 0$ which are valid near $x = 0$.

(OR)

(8) (a) Determine the singular points, Indicial polynomial and their roots for the equation $x^2 y^{11} + (x + x^2) y - y = 0$.

(b) Find all solutions Φ of the form $\phi(x) = |x|^r \sum_{k=0}^{\infty} c_k x^k$, $|x| > 0$, for the equation $x^2 y^{11} + xy^1 + (x^2 - 1/4)y = 0$.

Unit-V

(9) (a) State and prove Gronwall – Reid – Bellman inequality.

(b) Apply the method of successive approximation to the solutions of the IVP $u^1 = 1 + u^2$, $u(0) = 0$ and draw their graphs.

(OR)

(10)(a) State and prove Picards – Lindelof theorem.

(b) State comparison principle.

M.Sc.(Previous)DEGREE EXAMINATION,

MODEL PAPER

First Semester

Applied Mathematics

Paper IV – TOPOLOGY

Time : Three hours

Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

UNIT – I

- (1) (a) Define a metric space. Show that every non empty open set on the real line is the union of a countable disjoint class of open intervals.
(b) Show that a subset F of a metric space is closed if and only if its complement F' is open.

(OR)

- (2) (a) Define convergent sequence in a metric Space. State and prove Cantor's intersection theorem.
(b) Define a continuous function in a metric Space. Show that a mapping f of a metric space X into a metric space Y is continuous $\Leftrightarrow f^{-1}(G)$ is open in X for every open set G in Y .

UNIT – II

- (3) (a) Define a Topological space . Let X be a topological space.
If A is a subset of X , then show that
$$\bar{A} = \{ x / \text{each neighborhood of } x \text{ intersects } A \}$$

(b) Let X be a topological space and A be a subset of X . Then show that
(i) $\bar{A} = A \cup D(A)$ and (ii) A is closed iff A contains $D(A)$

(OR)

- (4) (a) State and Prove Lindelof's Theorem.
(b) Show that every separable metric space is second countable.

UNIT – III

- (5) (a) Define a compact space. Show that any closed subspace of a compact space is compact.
(b) State and prove Tychonoff's theorem.

(OR)

- (6) (a) Show that every sequentially compact metric space is compact.
(b) State and prove Ascoli's theorem.

UNIT – IV

- (7) (a) Define a hausdorff space. Show that the product of any non empty class of hausdorff spaces is a hausdorff space.
(b) Show that a one to one continuous mapping of a compact space onto a hausdorff space is a homeomorphism.

(OR)

- (8) (a) Show that every compact hausdorff space is normal.
(b) State and prove Urysohn's Lemma.

UNIT – V

- (9) (a) Define a connected space. Show that a subspace of the real line is connected \Leftrightarrow it is an interval.
(b) Show that any continuous image of connected space is connected.

(OR)

- (10) (a) let X be a topological space and A be a connected subspace of X . If B is a subspace of X such that $A \subseteq B \subseteq \bar{A}$, then show that B is connected.
(b) Define a totally disconnected space. Let X be a hausdorff space. If X has an open base whose sets are also closed, then show that X is totally disconnected.

M.Sc.(Previous)DEGREE EXAMINATION,
MODEL QUESTION PAPER

First Semester

Applied Mathematics

Paper V – DISCRETE MATHEMATICS

Time : Three hours

Maximum : 70 marks

Answer ALL questions. Each question carries 14 marks

UNIT – I

(1)(a) Show that $n^3 + 2n$ is divisible by n .

(b) Define a tautology. Show that the expression $((P \wedge \sim Q) \rightarrow R) \rightarrow (P \rightarrow (Q \vee R))$ is a tautology.

(OR)

(2)(a) Obtain DNF and CNF of the following formula

$$(\sim P \vee \sim Q) \rightarrow (P \leftrightarrow \sim Q).$$

(b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow Q) \wedge (Q \rightarrow S)$.

UNIT – II

(3)(a) Define state machine congruence. Let f be a state

homomorphism from the state machine $M=(S,I, \delta)$ onto the state machine $M_1=(S_1 ,I, \delta_1)$. Then show that there exists a state machine congruence on M such that M is isomorphic to M_1 .

(b) Let $M=(S, I, O, \delta, \theta)$ be an i/o machine and M_R be its reduced machine. If h is an i/o homomorphism from M onto M_1 , then show that there exists an i/o homomorphism g from M_1 onto M_R such that $f = goh$, where f is a natural homomorphism

(OR)

(4) Minimize the states of the following machine and write reduced machine.

states	δ		θ	
	0	1	0	1
1	2	1	1	
2	5	0	0	
3	5	1	1	
4	5	1	1	
5	1	1	1	
6	8	1	1	
7	8	1	1	
8	2	0	0	
	6	1	1	
	5	1	1	
	1	1	1	
	5	1	1	
	2	1	1	
	3	0	0	
	3	1	1	
	5	1	1	

UNIT - III

- (5)(a) Define atom and join-irreducible element in a Lattice.
 Show that every atom is join-irreducible
 (b) State and prove the distributive inequalities in Lattices.

(OR)

- (6)(a) State and prove De Morgan's laws in a Lattice
 (b) State and prove Representation theorem in a Boolean Algebra .

UNIT - IV

- (7)(a) Find DNF of the polynomial $x(y+z)' + (xy+z)'$.

(b) Define an ideal. Prove that an ideal M in a Boolean algebra B is maximal if and only if for any $b \in B$ either $b \in M$ or $b' \in M$ but not both.

(OR)

(8) Minimize the following Boolean polynomial using Quine- Mc Clusky method

$$wx'y'z + w'xy'z' + wx'y'z' + w'xyz + w'x'y'z' + wxyz + wx'yz + w'xyz' + w'x'yz'$$

UNIT – V

(9)(a) Draw the diagram for the following switching circuit

$$P = x_1(x_2(x_3+x_4)+x_3(x_5+x_6))$$

(b) Determine the symbolic representation of the circuit given by

$$P = (x_1+x_2+x_3)(x_1'+x_2)(x_1x_2+x_1'x_2)(x_2'+x_3)$$

(OR)

(10) Explain the central lighting system in a room and draw its switching circuit.